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ABSTRACT

This volume is the seventeenth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by second-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of five groups of activities. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. In this unit, multiplication is approached as repeated addition on a number line. In a second set of lessons, multiplication is considered in conjunction with arrays. Addition and multiplication are then compared, and simple fractions are introduced. A final review section is also included. (SD)

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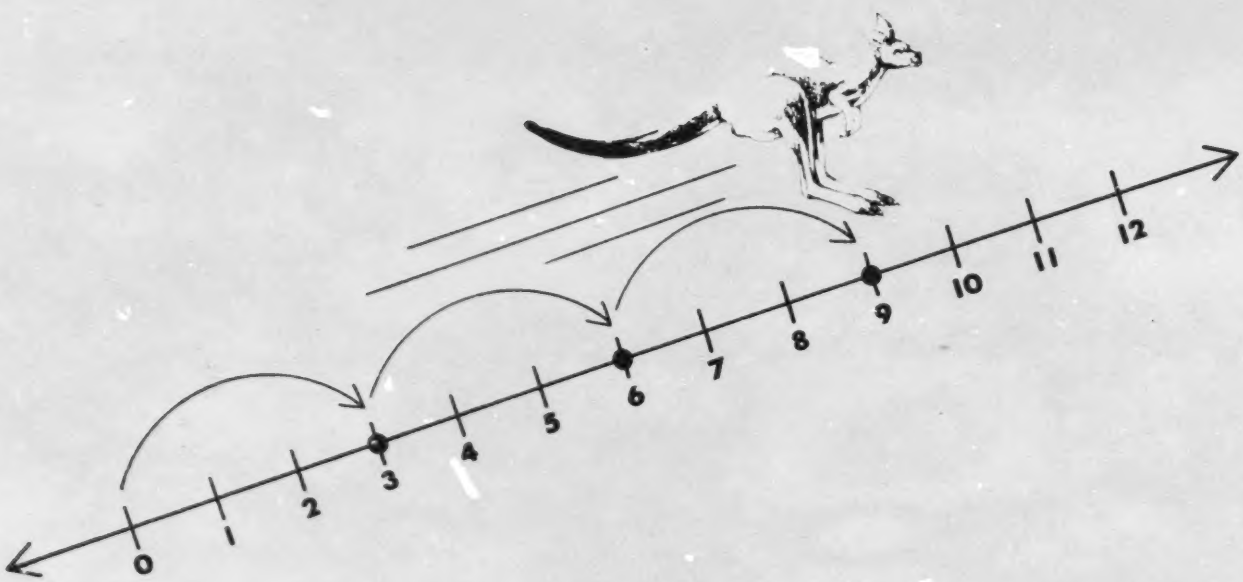
KANGAROOS & NUMBERS



SE 021 222

UNIT 2

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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1. WATCHING AND WONDERING
2. CURVES AND SHAPES
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6. NUMERATION
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FIRST GRADE

8. OBSERVING PROPERTIES
9. NUMBERS AND COUNTING
10. DESCRIBING LOCATIONS
11. INTRODUCING ADDITION AND SUBTRACTION
12. MEASUREMENT WITH REFERENCE UNITS
13. INTERPRETATIONS OF ADDITION AND SUBTRACTION
14. EXPLORING SYMMETRICAL PATTERNS

SECOND GRADE

15. INVESTIGATING SYSTEMS
16. NUMBERS AND MEASURING
17. INTRODUCING MULTIPLICATION AND DIVISION
18. SCALING AND REPRESENTATION
19. COMPARING CHANGES
20. USING LARGER NUMBERS
21. ANGLES AND SPACE

THIRD GRADE

22. PARTS AND PIECES
23. CONDITIONS AFFECTING LIFE
24. CHANGE AND CALCULATIONS
25. MULTIPLICATION AND MOTION
26. WHAT ARE THINGS MADE OF?
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INTRODUCING MULTIPLICATION AND DIVISION

KANGAROOS & NUMBERS

UNIT 17

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT
720 Washington Avenue S.E., Minneapolis, Minnesota 55455

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INTRODUCING MULTIPLICATION AND DIVISION

This unit was developed by MINNEMAST on the basis of experiences of the many teachers who taught an earlier version in their classrooms.

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Complete List of Materials for Unit 17

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	** Student Manuals	
1	demonstration number line (optional)	4,5,6
30	* number lines, 0-25	4,5,11
750	* counters	6,7,10,11, 14
30	pegboards and pegs (can substitute counters)	7
2	flannel boards (optional)	7,8,14
2-3	sets of objects for flannel board	7
24	fish or other objects for flannel board	8
30	*** sets of Minnebars	10
30	*** addition slide rules (if needed)	11
24	cookies or other objects for flannel board	14
30	scissors	15
3	18-inch strips of paper	16

*kit items as well as

**printed materials available from
Minnemath Center, 720 Washington Ave. S.E., Mpls., Minn. 55455

***available from The Judy Company,
310 North Second Street, Minneapolis, Minnesota 55401

INTRODUCTION

Multiplication is a mathematical operation that yields from any two real numbers a unique third number called the product. Multiplication of 3 and 4 yields the product 3×4. "3×4" is a name of a number that we also know as 12. It is also possible to think of "multiplication by 3" as an operation that transforms 4 into the product 3×4.

The method used to find the single numeral 12, which also names the product 3×4, is called a multiplication algorithm. There are several such algorithms available. Those that will be used at some time in the MINNEMAST program are:

1. Repeated addition using jumps on a number line.
2. Repeated addition using arrays or combinations of equivalent sets.
3. Related scales on parallel number lines.
4. Cartesian products. (This will be explained in a later second grade unit.)
5. A graphical method using a line of given slope.
6. A memorized table and computational rules.
7. A slide rule.

These algorithms differ in their generality. For example, algorithm 4 can be used only for the counting numbers 1, 2, 3, ..., whereas the graphical method of algorithm 5 can be applied to all real numbers (including, e.g. $\sqrt{2}$). In this unit we choose to use the first three algorithms. Some reasons for this choice are listed below.

1. All three algorithms have a strong connection with addition, with which the children are familiar.
2. Each algorithm can be given a simple physical interpretation.
3. As the need arises for multiplying other numbers such as fractions, the children need not unlearn the methods learned

in this unit but can modify them as they expand their concepts. All rules for computation can be reduced to addition, subtraction, multiplication, and division of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

4. Children need simple methods for finding the exact product of any whole numbers that they do not remember. The first three algorithms listed above provide these methods.

Different algorithms are presented so that the child will view the concept of multiplication in various ways. If a child is slow to understand one interpretation, he may understand it better after he sees another interpretation. Although the different algorithms are closely related, the relations will not be made explicit. However, the examples will give the children a feeling that underlying all of them is the mathematical idea of multiplication.

In this unit the children will work with only the numbers 0, 1, 2, 3, 4, 5, and 6. We restrict ourselves to these small numbers because they are easier for the children to work with, and at the same time they show the methods as fully as larger numbers would.

Why Multiplication is Introduced at this Time

You may be wondering why simple multiplication is introduced so early in the second grade materials, before the children have had much opportunity to learn systematic procedures for addition and subtraction. There are several reasons for our early consideration of multiplication.

First, the testing of this unit has shown that second graders can easily grasp the concepts presented here. In fact, many become familiar with simple multiplication even when it is not presented to them in their school work.

Second, it is possible to give the children a much better understanding of our positional system of numeration if they understand products. For example, they will use the idea that "375" means $(3 \times 100) + (7 \times 10) + 5$.

Third, it becomes possible to deal with many of the non-arithmetical second grade topics more meaningfully if the children have a concept of products. This is true of the material on scaling and representation, and on areas of rectangles.

Fourth, consideration of the question of why one might wish to multiply should help the development of an understanding of the structure of the real number system. This unit emphasizes the concept of multiplication rather than the computational techniques for finding products.

Introduction to Division

Later, when the children have had enough experience with multiplication to feel comfortable with it, we will encourage them to think of division as the inverse of multiplication. But we would like to give them a background for division now, while multiplication is still somewhat strange to them. We will introduce division in a manner which makes full use of the children's conceptions of dividing up a set of things into equivalent subsets. In this way we not only take advantage of the non-arithmetical meaning of the word "divide," but we also encourage an understanding of simple fractional parts, such as a half and a third. It will also be possible, using this approach, to give the children a feeling for the relation between simple fractions and division by the counting numbers, e.g., the relation between a half and division by two.

NOTES ON THE TEACHING OF THIS UNIT

It is planned that Unit 17, Introducing Multiplication and Division, and Unit 18, Scaling and Representation, be taught simultaneously. It is expected that on the average two periods a day will be used for the two units.

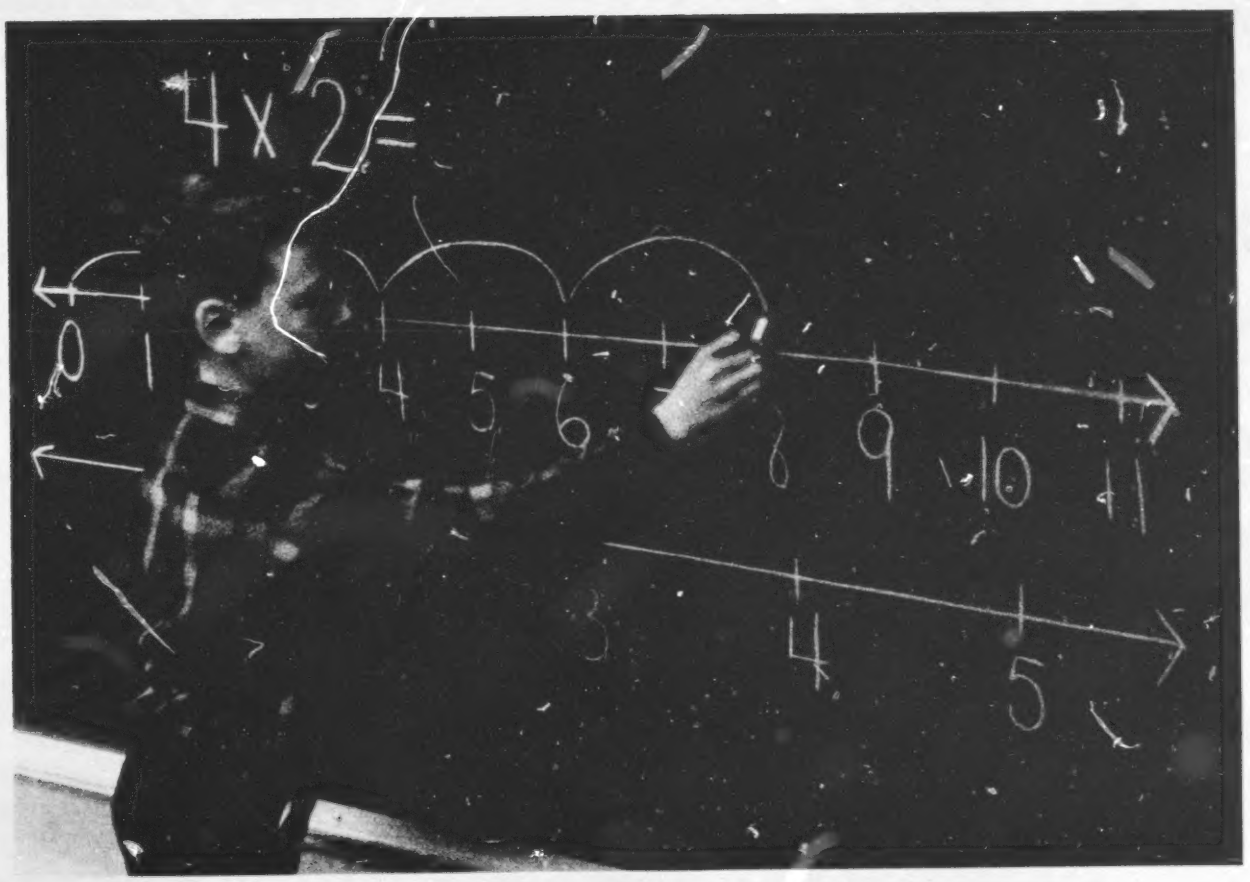
In the lessons, certain questions are shown in capital letters. These questions should be considered only as guides to the questions you will ask the children. You should modify them to fit the needs of your class. An answer shown in parentheses after the question may be only one of several appropriate replies.

Certain problems on the worksheets are starred. These are more difficult problems that you may wish to use for only part of your class.

The children should not be required to memorize the multiplication and division facts in second grade. They need to use them for a time and further develop their concepts of multiplication and division before they memorize them.

SECTION 1 MULTIPLICATON BY REPEATED ADDITION ON A NUMBER LINE

In this section it is shown that the product of two counting numbers may be found by repeated jumps on a number line. Two parallel number lines differing in scale are used to give a picture of the result of multiplying by a counting number. The concept of a pair of scaled number lines is also used in Unit 18, Scaling and Representation. Later, pairs of scaled number lines are used for division and for multiplication of numbers other than the counting numbers.



Lesson 1: REPEATED JUMPS ON THE NUMBER LINE

This lesson treats repeated addition of a counting number by repeating jumps of equal length on the number line. Multiplication is not yet mentioned. The term "addition equation" is used; however, you may use "addition sentence" if you wish.

MATERIALS

- Worksheets 1, 2, 3, 4 and 5

PROCEDURE

Read the following story to the children as an introduction to the worksheets. You may wish to use an overhead projector or copy Worksheet 1 with pictures large enough to be seen by the class when it is posted. There are kangaroos to cut out at the end of the unit if you wish to use them for demonstrations.

KANGAROO GAMES

Australia is a large country and it has many wide-open spaces where few people live. One of these wide-open spaces was chosen as a good spot for testing some new highway building machinery. Some men brought the equipment from far away. They set up tents, unpacked their supplies, and got the machines ready for testing the next morning. When they finished all their work they sat resting on camp chairs while the cook fixed their evening meal.

The men thought they were in a deserted spot. But they had a surprise waiting for them.

While they sat and rested, a curious family of kangaroos came close to watch them. There were a father, a mother, a small baby called Joey, and two partly-grown kangaroos named Chuck and Katie. The men and the kangaroos watched each other, until the cook called that supper was ready.

While the men were eating, the kangaroos slowly hopped about, eating grass and other plants. Occassionally they took long jumps.

One of the men, Jim, said, "I like to watch these animals. How can we keep them near the camp?"

"I made too many biscuits tonight," answered the cook. "If we put the extra ones out near the kangaroos, perhaps they will eat them and stay around hoping to get more."

Bill put 3 biscuits in one pile, 4 in another, and 2 in the last pile.

WHAT ADDITION EQUATION (or addition sentence) TELLS HOW MANY BISCUITS HE PUT OUT? (3 + 4 + 2 = 9)

4
5
4
3
2

The next day some of the men painted a number line on a long straight road. The men planned to use the number line in their machine tests, but the kangaroos thought it was painted especially for them to play on.

The first evening after the number line was painted the kangaroo family had jumping contests along the line. They found that Father Kangaroo jumped exactly 4 unit spaces each time. Mother Kangaroo jumped 3 unit spaces each time. Chuck made jumps of 2 unit spaces, and Katie made jumps of only 1 unit space. Joey, the baby of the family, couldn't jump at all, so Mother Kangaroo carried him wherever she went.

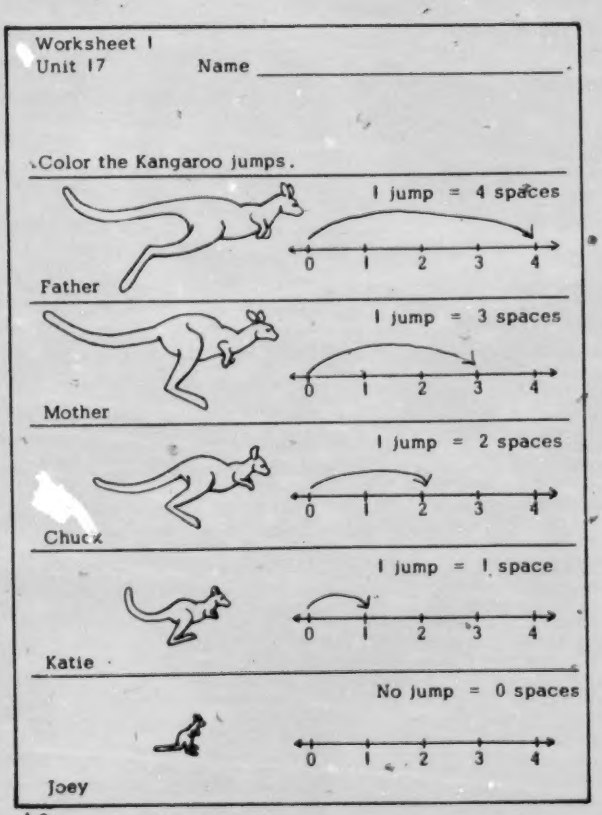
DOING THE WORKSHEETS

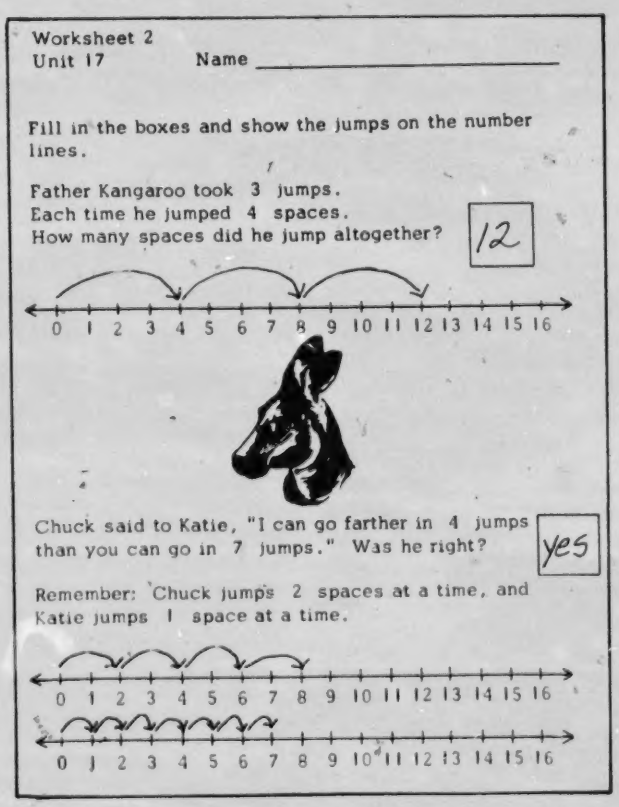
On Worksheet 1 the children are asked to show on number lines how far each kangaroo can go in one jump. They may refer to this worksheet as they work on the next few.

On Worksheet 2 they will find on number lines the total number of unit spaces covered by various combinations of kangaroo jumps.

On Worksheets 3, 4 and 5 the children will draw jumps and write addition sentences. You may wish to discuss the first problem or two in a manner similar to the following:

> "We can tell about the total number of unit spaces jumped by writing an addition equation. Look at Mother Kangaroo's trip on Worksheet 3. Who can write an addition equation telling about her trip? (3 + 3 + 3 + 3 = 12). We call addition such as this, where one number is added to itself several times, repeated addition."




Worksheet 2
Unit 17
Name
Fill in the boxes and show the jumps on the number lines.
Father Kangaroo took 3 jumps.
Each time he jumped 4 spaces.
How many spaces did he jump altogether?
12
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Chuck said to Katie, "I can go farther in 4 jumps than you can go in 7 jumps." Was he right?
yes
Remember: Chuck jumps 2 spaces at a time, and Katie jumps 1 space at a time.
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

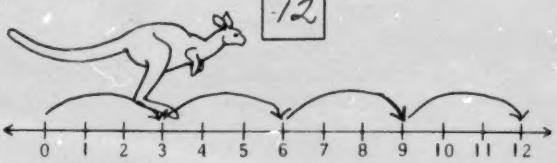
10

NOTE: In the first problem on Worksheet 5, Chuck Kangaroo takes two 2-space jumps, rests, and then takes three 2-space jumps. The addition sentence describing this situation can be written $(2 + 2) + (2 + 2 + 2) = 10$, where the parentheses show the grouping of the jumps. Unless a child suggests this form, simply use $2 + 2 + 2 + 2 + 2 = 10$.

Worksheet 3
Unit 17 Name _____

Show the jumps on the number lines and fill in the boxes.

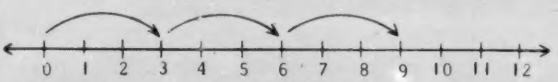
Mother Kangaroo took 4 jumps of 3 unit spaces each.
How far did she go? 12 spaces



Write an addition equation that tells about her jumps.

$3 + 3 + 3 + 3 = 12$

The next time her jumps looked like this:



Write an additon equation about these jumps.

$3 + 3 + 3 = 9$

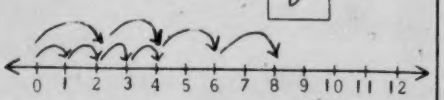
Worksheet 4
Unit 17 Name _____

Show the jumps on the number lines and fill in the boxes.

Katie and Chuck were sitting on the zero mark. Father told them they could each jump 4 times. Remember: Katie jumps 1 space at a time and Chuck jumps 2 spaces at a time.

How far did Katie jump? 4 spaces

How far did Chuck jump? 8 spaces



An addition equation that tells about Katie's jump is

$1 + 1 + 1 + 1 = 4$

An addition equation that tells about Chuck's jump is

$2 + 2 + 2 + 2 = 8$

Worksheet 5
Unit 17 Name _____

Chuck Kangaroo jumps 2 spaces at a time. He started at 0, took 2 jumps, rested, and then took 3 more jumps.

On what number did he stop? 10

Write an addition sentence about his jumps.

$2 + 2 + 2 + 2 + 2 = 10$

How far can Chuck go in 3 jumps? 6 spaces

Lesson 2: MULTIPLICATION AS REPEATED ADDITION ON A NUMBER LINE

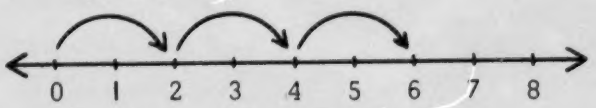
This lesson uses the number line to show that multiplication of counting numbers may be regarded as repeated addition. The terms "multiply" and "multiplication" are introduced. The term "product" need not be used in this lesson.

MATERIALS

- Worksheets 6, 7, 8 and 9

PROCEDURE

1. Ask a child to draw 3 steps of 2 units each on a chalkboard number line, beginning at zero.



Write $2 + 2 + 2 = 6$ on the board, saying that this tells about the steps on the number line. Ask:

HOW MANY TIMES WAS 2 ADDED ON THE NUMBER LINE? (3)

Explain that another way of saying this is "three times two" or "3 x 2". Thus, instead of the addition equation $2 + 2 + 2 = 6$, we can write the shorter multiplication equation $3 \times 2 = 6$. We multiply 3 times 2 to get 6.

2. Write $2 + 2 + 2 = 6$ and $3 \times 2 = 6$ side by side on the board and erase the previous marks on the number line.

Ask another child to draw 4 steps of 2 units each on the number line, beginning at zero. Again develop two sentences describing the steps:

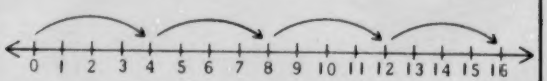
$2 + 2 + 2 + 2 = 8$ and $4 \times 2 = 8$

Read the multiplication sentence as "four times two equals eight." Write these equations under the other two.

At times you can also say "four two's" when referring to $2 + 2 + 2 + 2$ or 4×2.

3. Repeat the above procedure as needed to prepare the children for Worksheets 6, 7, 8 and 9. You might use examples leading to 2×4, 1×4, 4×1, 2×6, etc.

Worksheet 6
Unit 17 Name _____

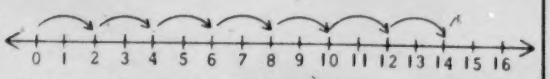


Write an addition equation about the steps on this number line.

$4 + 4 + 4 + 4 = 16$

Write a multiplication equation about the same steps.

$4 \times 4 = 16$



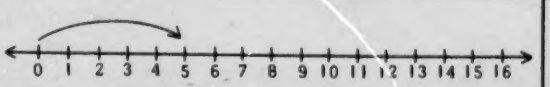
Write an addition equation about the steps on this number line.

$2 + 2 + 2 + 2 + 2 + 2 + 2 = 14$

Write a multiplication equation about the same steps.

$7 \times 2 = 14$

Worksheet 7
Unit 17 Name _____

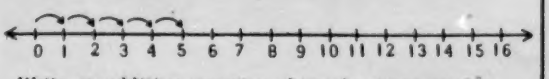


Write an addition equation about the steps on this number line.

$5 = 5$

Write a multiplication equation about the same steps.

$1 \times 5 = 5$



Write an addition equation about the steps on this number line.

$1 + 1 + 1 + 1 + 1 = 5$

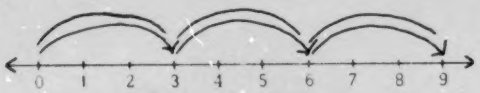
Write a multiplication equation about the same steps.

$5 \times 1 = 5$

Worksheet 8
Unit 17 Name_____

Draw blue jumps to show 3 + 3 + 3 = 9.

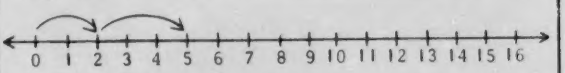
Draw red jumps to show 3 x 3 = 9.



Draw blue jumps to show 4 + 4 = 8.

Draw red jumps to show 2 x 4 = 8.

Worksheet 9
Unit 17 Name _____

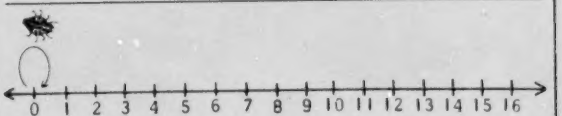


Write an addition equation about the steps on this number line.

2 + 3 = 5

Can you write a multiplication equation about these steps?

NO



A bug jumped up and came down just where he started.
He jumped 0 unit spaces.
Write the multiplication equation telling about this.

1 x 0 = 0

HINT: Think about these questions.
How many jumps did the bug make?
How long was each jump?

NOTE: Worksheet 8 shows an Australian tree frog.

After the children have completed the problems on Worksheet 8, ask them what they have discovered about the red and blue markings. (They are, of course, the same.)

Lesson 3: SHOWING MULTIPLICATION ON PARALLEL NUMBER LINES

Here a parallel number line method of showing multiplication is introduced. We will use the jumps of the Kangaroo family to develop two parallel number lines of different scales. These pairs will be called a 2-scale or a 3-scale or a 4-scale, etc. It is not intended that the students learn the meaning of the word "parallel". It is important they learn to read the products from the number lines.

Similar parallel scaled number lines are developed in Lesson 4 of Unit 18.

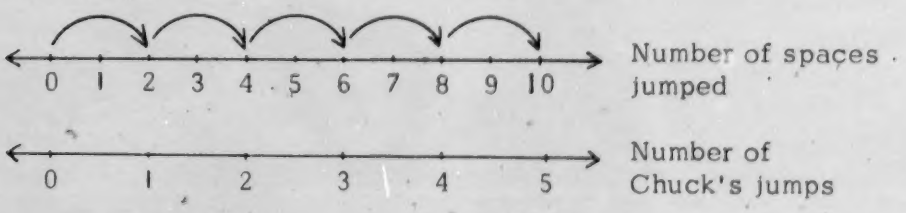
MATERIALS

- Worksheets 10, 11 and 12

PROCEDURE

Draw a number line on the chalkboard. Have one of your students show Chuck Kangaroo's jumps on the number line starting at 0.

Now draw a number line parallel to and below the first one. Label the top line "number of spaces jumped." Label the bottom line "number of Chuck's jumps" and number it accordingly.



Ask the following questions:

HOW MANY SPACES WILL CHUCK MOVE IN 3 JUMPS? (Locate 3 on the "Chuck's jumps" line, and look at the number directly above it on the "number of spaces" line. (6) This illustrates 3 x 2 = 6; however, at this time, do not write this multiplication sentence.)

HOW MANY SPACES WILL CHUCK MOVE IN 2 JUMPS?
(4)

Erase the arrows showing the jumps on the top line.

HOW MANY SPACES WILL CHUCK MOVE:

IN 1 JUMP? (2)

IN 0 JUMPS? (0)

IN 4 JUMPS? (8)

IN 5 JUMPS? (10)

Repeat the above questions until you are certain the children are finding the answers from the pair of parallel lines. Call this pair of lines a "2-scale."

You may suggest that one way to think of this pair of scaled number lines is that the upper line can be "stretched" until each part is 2 times as long, to get the lower line.

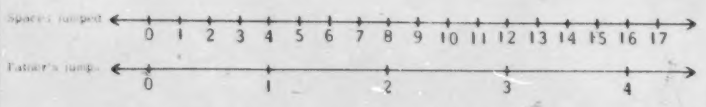
Repeat this procedure on your chalkboard by using Father Kangaroo (4-scale), Mother Kangaroo (3-scale), and Katie Kangaroo (1-scale).

After the above has been completed, the children should do Worksheets 10, 11 and 12. You may need to help the children get started, but encourage them to work as independently as possible. The starred problem (*) is for enrichment.

Finish the chart.
Find your answers from the number line.

Worksheet 12
Unit 17

Name _____

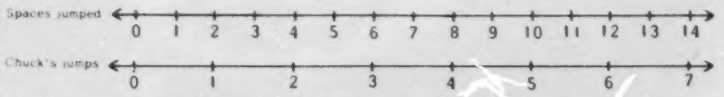


Spaces jumped	0	4	8	12	16
Number of Father's jumps	0	1	2	3	4

Finish the chart.
Find your answers from the number line.

Worksheet 10
Unit 17

Name _____

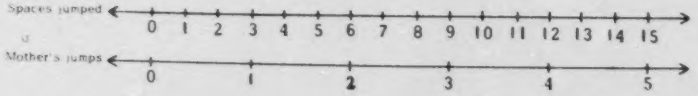


Spaces jumped	0	2	4	6	8	10	12	14
Number of Chuck's jumps	0	1	2	3	4	5	6	7

Finish the chart.
Find your answers from the number line.

Worksheet 11
Unit 17

Name _____



Spaces jumped	0	3	6	9	12	15	18
Number of Mother's jumps	0	1	2	3	4	5	*6

Lesson 4: SMALL MULTIPLICATION CHARTS

The term "product" is introduced in this lesson. Practice is given in recording products on charts.

MATERIALS

- demonstration number line or a number line drawn on the chalkboard
- number line (0 to 25) for each child
- Worksheets 13 and 14

PROCEDURE

Ask a child to show 2 steps of 3 unit spaces each on the chalkboard number line. Have another child write the multiplication sentence for this on the board. (2 x 3 = 6). Erase the marks showing the moves on the number line.

Use this procedure with other numbers to obtain the following table:

2 x 3 = 6

4 x 1 = 4

4 x 3 = 12

2 x 1 = 2

Introduce the word "product" to the class by using it in your discussion. When one number is multiplied by another the result is the product.

Explain that the products can be recorded in a shorter way on a multiplication chart. Draw and explain the following chart:

x	3	1
2	6	2
4	12	4

Use sentences such as, "The product of 4 and 1 is found where the 4-row intersects the 1-column." Trace the row and column with your fingers to show the location of the product.

CAN YOU FIND A CHART LIKE THIS ONE ON WORKSHEET 13? (Yes, in the upper left corner.)

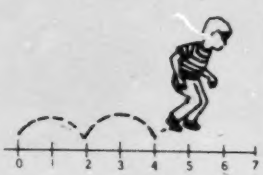
Have the children fill in the charts on Worksheet 13. They should use their number lines and work as independently as possible. After they have completed Worksheet 13, have them do Worksheet 14.

Worksheet 13
Unit 17 Name _____

×	3	1
2	6	2
4	12	4

×	2	5
4	8	20
1	2	5

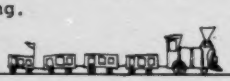
×	1	0	3
3	3	0	9
0	0	0	0



Worksheet 14
Unit 17 Name _____

Write a multiplication sentence for each problem.
Find the product on a chart on Worksheet 13.

Tom has a train that is 2 times as long as John's train.
John's train is 3 feet long.
How long is Tom's train?



2 × 3 = 6 feet

Bill has twice as many balls as Tim.
Tim has 3 balls.
How many balls does Bill have?

2 × 3 = 6

Ann is 4 times as old as Sue.
Sue is 2 years old.
How old is Ann?

4 × 2 = 8

Sally walks 3 times as far to school as Ruth does.
Ruth walks 3 blocks.
How far does Sally walk?

3 × 3 = 9

Lesson 5: MAKING A MULTIPLICATION CHART USING THE NUMBER LINE

In this lesson products from 0 x 0 to 6 x 6 are found by repeated addition on the number line and are recorded on a chart.

MATERIALS

- chalkboard number line -- either a demonstration number line or one drawn on the chalkboard, numbered to 36
- Worksheets 15 and 16
- number line to at least 25 for each child

PROCEDURE

1. Ask a child to show 2 steps of 3 unit spaces each on the chalkboard number line. Ask someone to write both a multiplication and an addition sentence on the chalkboard describing the steps on the number line. (3 + 3 = 6 and 2 x 3 = 6).

   Ask another child to show 3 steps of 3 unit spaces each. Have these equations written directly below the first equations.

   3 + 3 + 3 = 9 and 3 x 3 = 9

   THIS TIME WE WILL TAKE JUST ONE STEP OF 3 UNIT SPACES. CAN SOMEONE SHOW THIS ON THE NUMBER LINE AND WRITE THE EQUATIONS ABOVE THE EQUATIONS ALREADY WRITTEN? (3 = 3 and 1 x 3 = 3)

   NOW CAN SOMEONE GUESS HOW TO WRITE A MULTIPLICATION EQUATION WHEN NO 3-STEPS ARE TAKEN? (0 x 3 = 0)

   WRITE THE EQUATION ABOVE THE MULTIPLICATION EQUATION FOR 1 x 3.

The chalkboard chart should now look like this:

	0 x 3 = 0
3 = 3	1 x 3 = 3
3 + 3 = 6	2 x 3 = 6
3 + 3 + 3 = 9	3 x 3 = 9

This chart could have been constructed by starting at 0 x 3 = 0, but it will be less confusing for the children to start with the more obvious 2 x 3 = 6. Continue the chart to 6 x 3 = 18. It will look like this:

	0 x 3 = 0
3 = 3	1 x 3 = 3
3 + 3 = 6	2 x 3 = 6
3 + 3 + 3 = 9	3 x 3 = 9
3 + 3 + 3 + 3 = 12	4 x 3 = 12
3 + 3 + 3 + 3 + 3 = 15	5 x 3 = 15
3 + 3 + 3 + 3 + 3 + 3 = 18	6 x 3 = 18

Have a child read the column of products from the chalkboard chart. Elicit the response that when the child reads these products in order he is counting by threes. It is useful to mark the products on the number line. Bring out the fact that each product can be obtained by adding 3 to the preceding product.

Point out to your class that the products can be recorded in a shorter way on a multiplication chart such as the one on Worksheet 15. Help the children complete the column corresponding to multiplication of the numbers 0 through 6 times 3.

2. In a manner similar to that above have the following chart made on the chalkboard.

	0 x 6 = 0
6 = 6	1 x 6 = 6
6 + 6 = 12	2 x 6 = 12
6 + 6 + 6 = 18	3 x 6 = 18
6 + 6 + 6 + 6 = 24	4 x 6 = 24
6 + 6 + 6 + 6 + 6 = 30	5 x 6 = 30
6 + 6 + 6 + 6 + 6 + 6 = 36	6 x 6 = 36

The children should follow the board work on their own number lines until the products become greater than the numbers shown on the number lines. The products should be recorded on Worksheet 15.

3. The children should now complete Worksheet 15. They may use the completed Worksheet 13 or their individual number lines.

4. Tell the children that because it is easier to find numbers when they are in counting order, multiplication charts are usually made like the one on Worksheet 16. Have the children cut out the columns from the chart on Worksheet 15 and paste them over the corresponding column on Worksheet 16. (If you wish, you may have the rows simply re-copied in order.) Now, both rows and columns of the multiplication chart should be in order of increasing integers.

   Do not require the students to memorize the multiplication facts. Let them refer to this chart whenever it is necessary to find a product. The patterns in the chart will be discussed later. However, a child who notices some of them now should be encouraged in his observations.

NOTE: Have the children save Worksheet 16 for use in future lessons.

Worksheet 15
Unit 17 Name _____

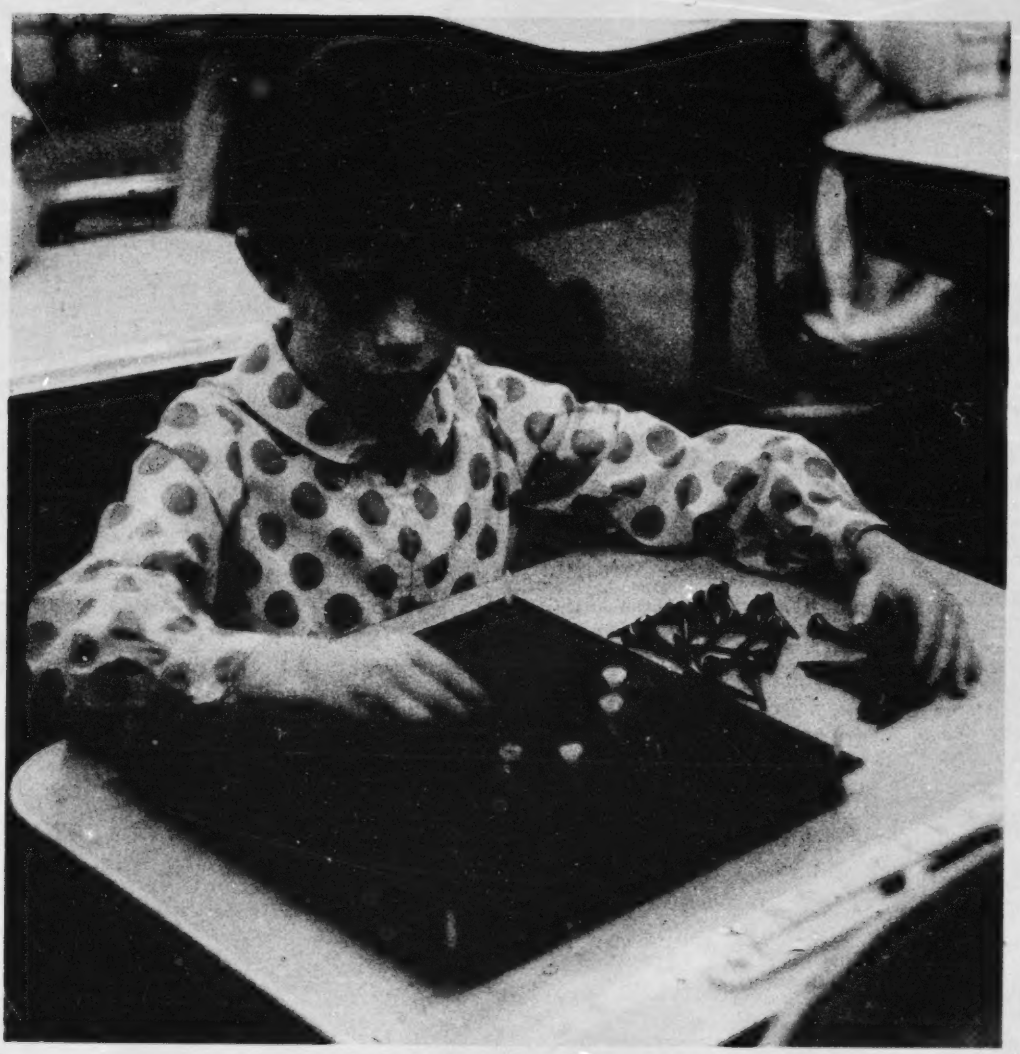
×	3	2	1	5	4	0	6
0	0	0	0	0	0	0	0
1	3	2	1	5	4	0	6
2	6	4	2	10	8	0	12
3	9	6	3	15	12	0	18
4	12	8	4	20	16	0	24
5	15	10	5	25	20	0	30
6	18	12	6	30	24	0	36

Worksheet 16
Unit 17 Name _____

×	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	8	10	12
3	0	3	6	9	12	15	18
4	0	4	8	12	16	20	24
5	0	5	10	15	20	25	30
6	0	6	12	18	24	30	36

SECTION 2 ARRAYS AND MULTIPLICATION

In this section the children find products by counting objects in arrays. It is shown that this method is very closely related to using repeated jumps on a number line, and of course that it gives the same results.



Lesson 6: MULTIPLICATION USING ARRAYS

In this lesson products are found by counting the objects in rectangular arrays. This is shown to be the same as joining equivalent sets of unit spaces on a number line.

MATERIALS

- demonstration number line
- 20 counters to place on the number line (put sticky putty or pieces of masking tape on paper counters)
- Worksheets 17 and 18

PROCEDURE

Put 4 counters on the desk of each of 5 children.

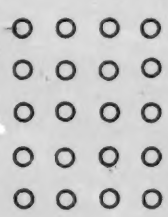
HOW MANY CHILDREN HAVE COUNTERS? (5)

HOW MANY COUNTERS DOES EACH ONE HAVE? (4)

HOW CAN WE FIND OUT HOW MANY COUNTERS THERE ARE ALTOGETHER? (Count or add. If a child says multiply 5 x 4, change the question to "Can you show this on the number line?")

Have each of the 5 children in turn place his counters on the number line. In discussion bring out the following facts: We joined 5 sets of 4 counters each and found we had 20 counters. The addition equation for this is $4 + 4 + 4 + 4 + 4 = 20$. The multiplication equation is $5 \times 4 = 20$. (You can think of this equation as saying that 5 times we had 4 objects.) Each counter corresponded to a unit space on the number line, so we also joined 5 sets of 4 spaces each.

Have each of the 5 children pick up his 4 counters. This time have each put down his counters as one row of a 5 x 4 array:

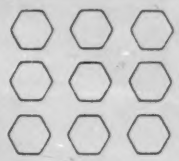


Explain to the class that this is a neat, easy way to show 5 sets of 4 counters each. The arrangement has 5 horizontal rows and 4 vertical columns. It is called a rectangular array. The total number of counters in the array is $5 \times 4 = 20$.

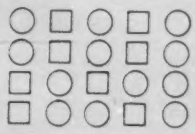
Repeat the above procedure with other small integers, if you wish, and then proceed with Worksheets 17 and 18.

Worksheet 17
Unit 17 Name _____

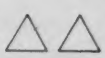
Fill in the boxes.



Number of rows 3
Members in each row 3
Members in the array 9
Multiplication equation 3 x 3 = 9



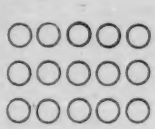
Number of rows 4
Members in each row 5
Members in the array 20
Multiplication equation 4 x 5 = 20



Number of rows 1
Members in each row 2
Members in the array 2
Multiplication equation 1 x 2 = 2

Worksheet 18
Unit 17 Name _____

Fill in the boxes.

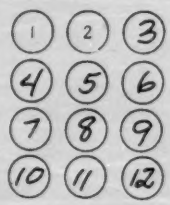


Number of rows 3
Members in each row 5
Multiplication equation 3 x 5 = 15

Number of rows 5
Members in each row 3
Multiplication equation 5 x 3 = 15

Finish numbering the arrays and finish the equations.

2 x 4 = 8



4 x 3 = 12

Lesson 7: MAKING ARRAYS

More practice is given here in making arrays to illustrate products.

MATERIALS

- individual pegboards with pegs, or 25 counters per child
- flannel board with sets of objects (optional)
- Worksheets 19 and 20

PROCEDURE

The problems on Worksheet 19 can be solved by making appropriate arrays and counting the members. Suggest that the class make the arrays on a pegboard or with counters. You may wish to have volunteers show their arrays on a flannel board. Perhaps some children would like to make up more entries for the parade.

The terms "row" and "column" may need to be reviewed. Sometimes we use the phrase "number in each row" in place of the equivalent "number of columns."

The game on the next page may be played after Worksheet 19 has been completed.

Worksheet 19
Unit 17 Name _____

Write a multiplication equation for each problem.

A band leads a parade.
There are 5 rows of players in the band.
There are 5 columns of players in the band.
How many players are in the band?

$5 \times 5 = 25$

Some scouts carry flags in the parade.
There are 2 rows of scouts.
There are 4 columns of scouts.
How many scouts are there?

$2 \times 4 = 8$

Next are some men on horses.
There are 5 rows of horses.
There are 3 columns of horses.
How many horses are there?

$5 \times 3 = 15$

Array Game

This game is described for an individual player. However, it may be played by partners or teams. The game may also be varied by using pegs and pegboards in place of tiles.

A player takes a number of square tiles — the number is determined by choice, by drawing a slip of numbered paper, or by taking a handful of tiles. He makes as many different rectangular arrays as he can from the tiles, but he must use every tile each time. The results are recorded on Worksheet 20, or on a similar record sheet. For example, from 15 tiles, a player could make arrays of 1 x 15, 3 x 5, 5 x 3, and 15 x 1.

The players should soon see that the arrays occur in pairs such as 2 x 3 and 3 x 2. They should also discover that many more rectangular arrays can be made from certain numbers of tiles than from others. For example, eight different arrays can be made from 24 tiles and only two arrays can be made from 23 tiles.

Worksheet 20
Unit 17 Name _____

I made ☐ rectangular arrays from ☐ tiles.
The arrays were:

I made ☐ rectangular arrays from ☐ tiles.
The arrays were:

I made ☐ rectangular arrays from ☐ tiles.
The arrays were:

The players may devise their own scoring rules. One possible way to score is to give one point for each array and to total the points for five trials. The winner is the one with the greatest score.

Lesson 8: COMMUTATIVITY OF MULTIPLICATION

In this lesson the commutative (or order) principle of multiplication is presented. This principle states that the factors may be taken in either order; for example, $3 \times 4 = 4 \times 3$. The children should understand and use the commutative principle, but they need not use the term.

MATERIALS

- 2 flannel boards or 2 boards, pieces of cardboard, or magazines
- 24 shapes for the boards — preferably fish (there is a pattern at the back of this manual)
- Worksheet 21

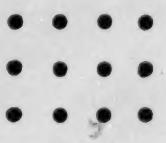
PROCEDURE

Place two flannel boards before the class. (Or place two boards, pieces of cardboard or magazines on a table.) Have available 24 simple fish shapes. Read or tell the following to the children:

Seven children, on a picnic with their parents, decided to go fishing. The 3 boys went in one boat

and the 4 girls in another boat. When they came in for lunch, the boys shouted to the girls, "We have more fish than you do. Each of us caught 4 fish." The girls called, "You do not. Each of us caught 3 fish, and there are more of us." Who was right?

Place the 4 fish for one boy in a row on one flannel board (or on one of the boards or magazines) saying that you are going to make a picture of the number of fish caught by the boys. Place under this row the rows of fish for the other two boys, producing a rectangular array.



Ask a child to count the total number of fish and write the corresponding multiplication equation on the blackboard. ($3 \times 4 = 12$, not $4 \times 3 = 12$)

Similarly on the other flannel board (or board or magazine) make a rectangular array showing the girls' fish.

Again have a child count the total number of fish and write the corresponding multiplication equation. (This time, $4 \times 3 = 12$)

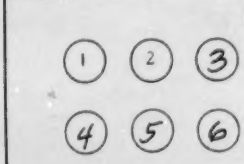
Someone should by now have volunteered the information that the boys and girls caught the same number of fish. If not, bring this out, and then ask if the arrays for the boys and girls look alike. A child will probably suggest rotating one array to produce the other array. Actually rotate one of the

flannel boards to show this. It does not matter whether we have 3 rows of 4 fish or 4 rows of 3 fish; the total is the same: $3 \times 4 = 4 \times 3$.

The children should now be ready for Worksheet 21 where more examples of the commutativity of multiplication are given.

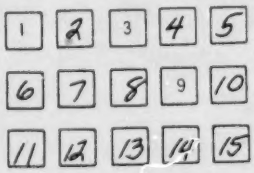
Worksheet 21
Unit 17 Name: _____

Finish numbering the arrays.
Finish the equations.

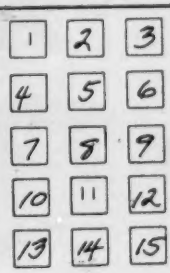


2 x 3 = 6

3 x 2 = 6



3 x 5 = 15



5 x 3 = 15

Lesson 9: MAKING A MULTIPLICATION CHART USING ARRAYS

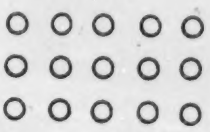
In this lesson, as in Lesson 5, a multiplication chart is made up to 6 x 6. This time the products are found by using arrays. This method differs from, but is closely related to, the method using repeated addition on the number line. The charts will be seen to be identical.

MATERIALS

- Worksheets 22, 23, 24 and 25

PROCEDURE

Tell the class that they will make another multiplication chart by counting the number of members in arrays. As an example, ask a child to draw and count a 3 x 5 array.



All the children should enter the product 15 in the box where the 3-row and the 5-column intersect on Worksheet 25.

Ask if they should draw a 5 x 3 array to find the product 5 x 3. If the answer is no, agree and point out that only half of the products in the chart need to be found because the other half just have the order of the factors reversed. (Note that this is the first use of the term "factor." Introduce it casually by repeated example.) If the answer above is yes, agree that this would give the product 5 x 3, but ask if there is an easier way. This should bring a suggestion to change the order of the factors. Someone may also correctly suggest viewing the 3 x 5 array from the side. Have the children enter the product 15 in the box where the 5-row and 3-column intersect.

Now tell the class that you know another short cut for finding the products. (This "short cut" is primarily introduced to give practice in recognizing relations among numbers.) Instead of

drawing and then counting arrays, we can number the members as we go along.

Have each child complete numbering the top row of the first array on Worksheet 22. This numbering shows that 1 x 4 = 4. Have him write this equation beside the array.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24

1 x 4 = 4
2 x 4 = 8
3 x 4 = 12
4 x 4 = 16
5 x 4 = 20
6 x 4 = 24

Continue by having each child number the next row of four boxes and write the appropriate equation. Continue building up the array until it has six rows and 24 boxes.

Discuss this array in class. One fact that should be brought out is that each product can be obtained by adding 4 to the

Worksheet 22
Unit 17 Name _____

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24

1 x 4 = 4
2 x 4 = 8
3 x 4 = 12
4 x 4 = 16
5 x 4 = 20
6 x 4 = 24

1	2
3	4
5	6
7	8
9	10
11	12

1 x 2 = 2
2 x 2 = 4
3 x 2 = 6
4 x 2 = 8
5 x 2 = 10
6 x 2 = 12

Worksheet 23
Unit 17 Name _____

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

1 x 5 = 5
2 x 5 = 10
3 x 5 = 15
4 x 5 = 20
5 x 5 = 25
6 x 5 = 30

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18

1 x 3 = 3
2 x 3 = 6
3 x 3 = 9
4 x 3 = 12
5 x 3 = 15
6 x 3 = 18

preceding one, hence reading the products in order is counting by fours.

Enter the products in the chart on Worksheet 25 in the 4-column. Also complete the 4-row by using the commutative principle. (The children will not use the word, just the idea.)

Have the children complete Worksheets 22, 23, 24 and 25. For products where one factor is zero, explain that, for example, 0×4 can be thought of as the number of members in an array of zero rows of 4 each, which contains no members. Similarly, one can think of 5×0 as the number of members in an array of 5 rows each containing zero members.

When the chart on Worksheet 25 is completed, have the children compare it with Worksheet 16. If no errors have been made the charts will be identical. Although the mechanics of obtaining the charts are quite different, the underlying concept in both

Worksheet 24
Unit 17 Name_____

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

1 x 6 = 6
2 x 6 = 12
3 x 6 = 18
4 x 6 = 24
5 x 6 = 30
6 x 6 = 36

1
2
3
4
5
6

1 x 1 = 1
2 x 1 = 2
3 x 1 = 3
4 x 1 = 4
5 x 1 = 5
6 x 1 = 6

Worksheet 25
Unit 17 Name_____

×	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	8	10	12
3	0	3	6	9	12	15	18
4	0	4	8	12	16	20	24
5	0	5	10	15	20	25	30
6	0	6	12	18	24	30	36

methods is the use of repeated addition to obtain products of certain positive integers. This fact should be discussed with your class to the extent you judge worthwhile.

Have each child save his copy of Worksheet 16 or Worksheet 25 for future use.

Clap Game

This game may be played during or after Lesson 9. Ask the children to count aloud and clap on the multiples of three (for example). They would say, "One, two, three (clap), four, five six (clap)," etc.

Buzz Game

This is a more difficult version of the above. Ask the children to count aloud, saying "Buzz" instead of the multiples of three (for example). They would say, "One, two, buzz, four, five, buzz," etc.

Lesson 10: AREAS OF RECTANGLES WITH INTEGRAL LENGTHS

The concept of area has been developed in kindergarten and first grade units. Here we use the number of square units included in a rectangle with sides of integral length as a measure of the area included in the rectangle. We do not develop the rule, area equals length times width.

This lesson uses the idea of a floor plan. The children will be familiar with this idea from their lessons in Scaling and Representation.

MATERIALS

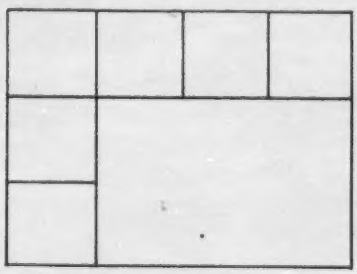
-- for each child --

- about 20 squares (e.g., unit Minnebars, square blocks, square counters, tiles)
- Minnebars
- Worksheets 26, 27, 28 and 29

PROCEDURE

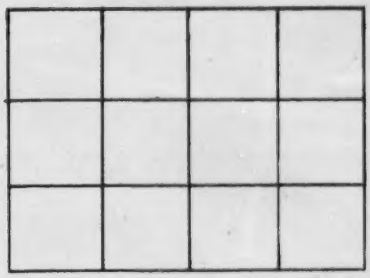
Read or tell the following to the children:

> The men in the Australian machine testing camp were constructing a temporary building. They planned to make the floor of square pieces of plywood and they wanted to know how many plywood squares they would need. Bill measured the floor. He found that exactly 3 squares fit across the end of the floor and exactly 4 squares fit along the side of the floor. A small-scale floor plan showing this would look like this:



(Sketch on board.)

Ask each child to use his squares to make a plan of the floor showing how the plywood squares would cover it.



HOW MANY ROWS ARE THERE? (3)

HOW MANY SQUARES ARE IN EACH ROW? (4)

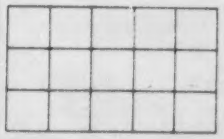
HOW MANY SQUARES ARE THERE ALTOGETHER? (12)

WRITE A MULTIPLICATION EQUATION THAT TELLS THE NUMBER OF SQUARES. (3 x 4 = 12)

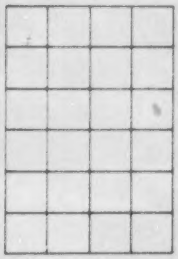
Tell the class that the number of squares is a measure of the area of the floor.

Repeat the above activity with other counting numbers, if you wish, before going on to the worksheets. Suggest that the children check their answers on Worksheet 26 by fitting Minnebars onto the diagrams and counting the unit squares.

Worksheet 26
Unit 17 Name_____



Number of rows 3
Number of columns 5
This is a 3 x 5 rectangle.
The area is 15 squares.

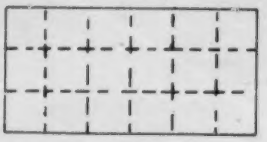


Number of rows 6
Number of columns 4
This is a 6 x 4 rectangle.
The area is 24 squares.

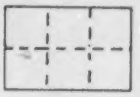
Number of rows 2
Number of columns 6
This is a 2 x 6 rectangle.
The area is 12 squares.

Worksheet 27
Unit 17 Name_____

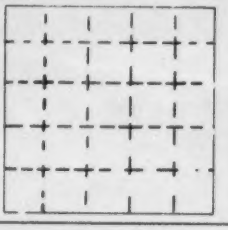
Place Minnebars inside the rectangle to find the area. Write a multiplication sentence about the area.



3x6=18



2x3=6



5x5=25

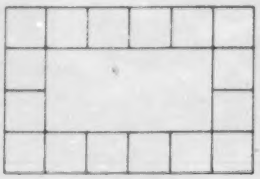
Worksheet 28
Unit 17 Name_____

1. Sandy wants to tile the floor of her playhouse. She placed tiles along 2 sides of the floor. You show the tiles by placing Minnebars on the picture of the floor.

How many tiles will she need for the whole floor? 18
Write a multiplication sentence telling the number of tiles.

3 x 6 = 18

2. A border of a flower garden is made of square blocks. Here is a scaled-down picture.



The area of the border is 16 square blocks.

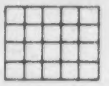
The area of the garden is 8 square blocks.

Worksheet 29
Unit 17 Name_____

Billy had a 4 x 2 rectangular array of blocks. Fred had one that was 4 x 3. George had one that was 4 x 5.

Billy's

Fred's



George's

How many blocks did Billy have? 8

How many blocks did Fred have? 12

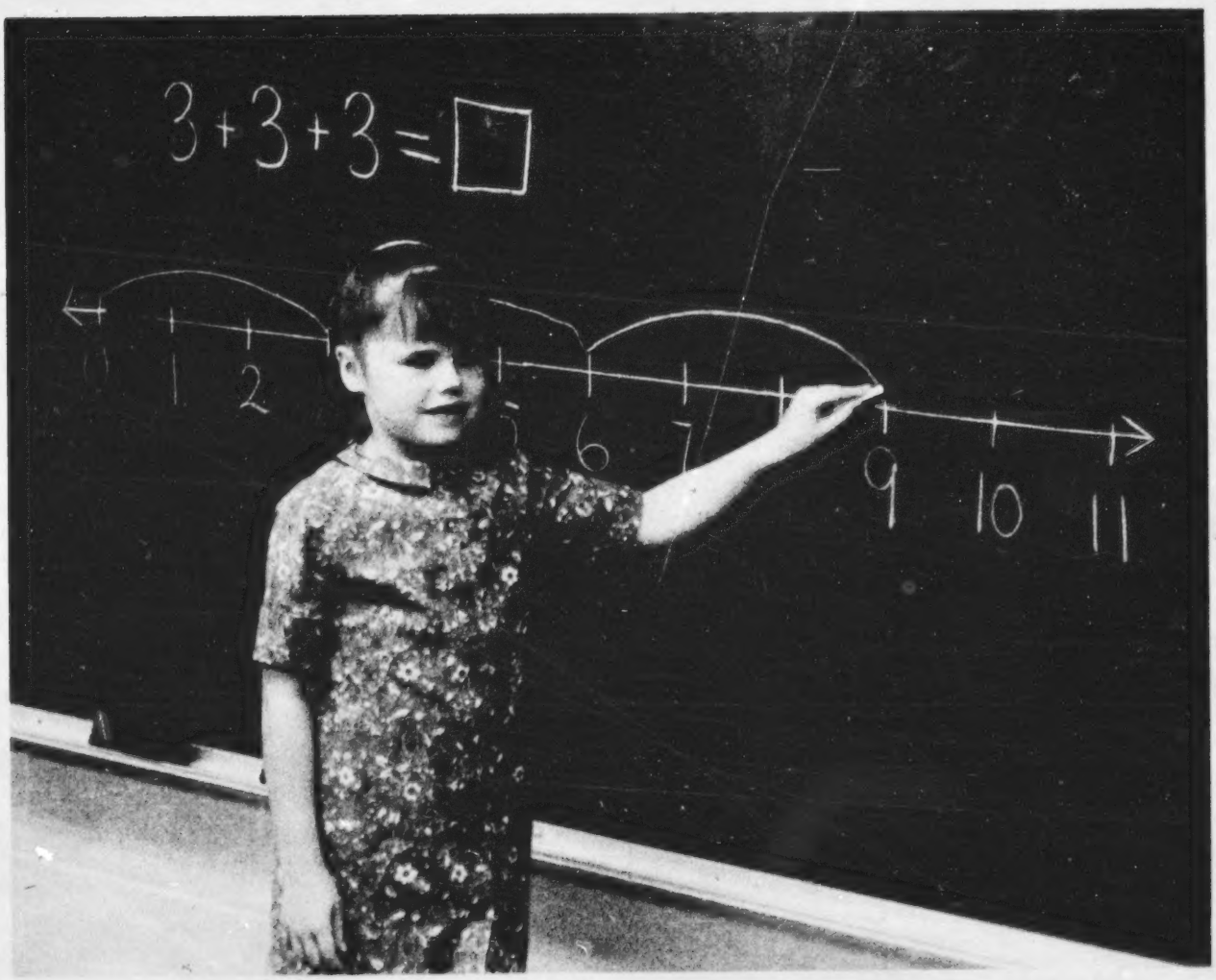
How many blocks did George have? 20

If Fred and Billy put their blocks together, would they have more blocks than George?

NO

SECTION 3 COMPARING ADDITION AND MULTIPLICATION

Addition and multiplication are two different mathematical operations. This may be difficult for the children to see until they are able to find products by an algorithm that does not directly involve repeated addition. However, this section gives the children a start toward understanding the concept by comparing the results of adding and multiplying the whole numbers from 0 to 6.



Lesson 11: MAKING AN ADDITION CHART

In this lesson the children are asked to complete a 6 x 6 addition chart. This chart will be used in the next lesson. Making the chart will also provide a review of ways of finding sums.

MATERIALS

- Worksheet 30
- addition aids such as number lines, counters, and addition slide rules

PROCEDURE

Have each child complete Worksheet 30, which is an addition chart with sums from 0 + 0 to 6 + 6. There will probably be children who fill in the chart by memory or by a simple counting procedure. However, it is expected that there will be other children who will need to use aids such as number lines, counters, and addition slide rules. Discuss with them the idea that only half of the sums need to be determined and the other half can be recorded by using the commutative principle. (However, don't use the word with the children). For example, after determining that 5 + 6 = 11, one immediately knows that 6 + 5 = 11.

Do not require the class to memorize this chart. The objective here is to have the chart correctly completed so that it can be used in the next lesson.

Worksheet 30
Unit 17 Name_____

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	7
2	2	3	4	5	6	7	8
3	3	4	5	6	7	8	9
4	4	5	6	7	8	9	10
5	5	6	7	8	9	10	11
6	6	7	8	9	10	11	12

Lesson 12: COMPARING ADDITION AND MULTIPLICATION

A comparison of a multiplication chart and an addition chart is made in this lesson.

MATERIALS

- completed copies of Worksheet 30 and Worksheet 16 or 25

PROCEDURE

1. Each child should have on his desk an addition and a multiplication chart. Ask questions such as these:

   DOES IT MATTER WHETHER YOU USE WORKSHEET 16 OR 25 FOR YOUR MULTIPLICATION CHART? (No, we saw they are the same.)

   ARE THE NUMBERS ON THE TWO CHARTS ON YOUR DESK THE SAME? (No.)

   WHAT IS THE NUMBER IN THE 2-ROW AND 4-COLUMN IN YOUR ADDITION CHART? (6)

   WHAT IS ANOTHER NAME FOR IT? (2 + 4)

   WILL SOMEONE WRITE THE ADDITION EQUATION ON THE BOARD? (2 + 4 = 6 or 6 = 2 + 4)

   WHAT IS THE NUMBER IN THE 2-ROW and 4-COLUMN IN YOUR MULTIPLICATION CHART? (8)

   WHAT IS ANOTHER NAME FOR IT? (2 x 4)

   WILL SOMEONE WRITE THE MULTIPLICATION EQUATION ON THE BOARD? (2 x 4 = 8 or 8 = 2 x 4)

Point out the different symbols used for the two operations and the different results of the operations. If you wish, repeat the questions with other numbers.

2. Have the children look for patterns in the charts. They should notice some of the following patterns, although they may not describe them in these words. They may notice other patterns also.

Each chart is symmetric about the diagonal that goes from the upper left to the lower right because both operations are commutative. In the multiplication chart the rows and columns involve "skip counting"; for example, the 3-column reads 0, 3, 6, 9, 12, 15, 18 where each term is found by adding 3 to the preceding term. The terms in a row or column in the addition chart are in counting order; for example, the 3-column reads 3, 4, 5, 6, 7, 8, 9. The ascending diagonals on the addition chart have identical terms; the descending diagonals are either odd or even numbers in order.

Lesson 13: APPLYING MULTIPLICATION AND ADDITION

The worksheets with this lesson have word problems that provide practice in applying multiplication and addition.

MATERIALS

- Worksheets 31, 32, 33 and 34

PROCEDURE

This lesson is entirely presented on the worksheets. Some children may wish to manipulate counters to show the problems.

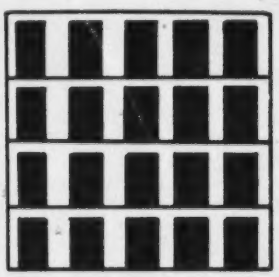
Worksheets 31 and 32

Each problem on Worksheets 31 and 32 is most naturally illustrated either by joining intervals on the number line or by arrays. However, these need not be the only methods used, nor should a child be required to use a particular method.

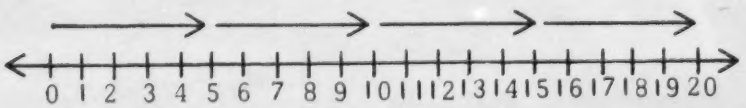
The first problem on Worksheet 31 can be used to illustrate some correct approaches to a solution.

A bookshelf has 4 shelves. There are 5 books on each shelf. How many books are there?

(a) Draw an array of books on shelves and count. Write the multiplication equation. $4 \times 5 = 20$.



(b) Show the books on a number line by some symbol; you might wish to use the following:



The appropriate equation is either $4 \times 5 = 20$ or $5 + 5 + 5 + 5 = 20$.

(c) Use a 5-scale to show $4 \times 5 = 20$.

(d) Add the number of books directly as $5 + 5 + 5 + 5 = 20$ or $4 \times 5 = 20$.

Some children, immediately on reading the problem, say or write $4 \times 5 = 20$. Naturally this is very good, but ask them to illustrate the solution with a diagram this time and a few other times.

Neither the number lines nor the arrays need to be precisely drawn with rulers.

Worksheet 31
Unit 17 Name _____

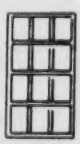
1. A bookshelf has 4 shelves.
There are 5 books on each shelf.
How many books are there?

$4 \times 5 = 20$

2. Mary has a garden with one row of beans, one row of peas, and one row of corn.
There are 6 plants in each row.
How many plants does she have?

$3 \times 6 = 18$

3. John's room has a window with small panes.
There are 4 rows of 3 panes each.
How many panes are there?



$4 \times 3 = 12$

Worksheet 32
Unit 17 Name _____

1. Bill measured the length of the table with a 6 inch ruler.
He found the table was as long as 4 ruler lengths.
How many inches long was the table?

$4 \times 6 = 24$

2. Jane poured 3 pans of milk into a large bowl.
Each pan contained 2 cups of milk.
How many cups of milk were in the bowl?

$3 \times 2 = 6$

3. Three cats each had 4 kittens.
How many kittens were there altogether?

$3 \times 4 = 12$

Worksheets 33 and 34

Some of the problems on these worksheets are answered by an addition equation and some by a multiplication or repeated addition equation. If a child correctly uses a repeated addition equation, approve it, but ask if he can also write the corresponding multiplication equation. Deciding which operation to use may be difficult for some children, but it is basic to the understanding of the operations.

Worksheet 33
Unit 17 Name_____

1. A store has 4 red bicycles and 3 blue bicycles.
   How many red and blue bicycles does it have?

   $4 + 3 = 7$

2. The store has 3 bicycle racks.
   Each can hold 4 bicycles.
   How many bicycles can the racks hold?

   $3 \times 4 = 12$

3. A truck brings 5 new red bicycles and 3 new green bicycles to the store.
   How many new bicycles does it bring?

   $5 + 3 = 8$

Worksheet 34
Unit 17 Name_____

1. 9 boys each have no mittens.

   How many mittens do they have altogether?

   $9 \times 0 = 0$

2. Tom has 6 toy cars.
   Bill has 2 toy cars.

   How many do Tom and Bill have together?

   $6 + 2 = 8$

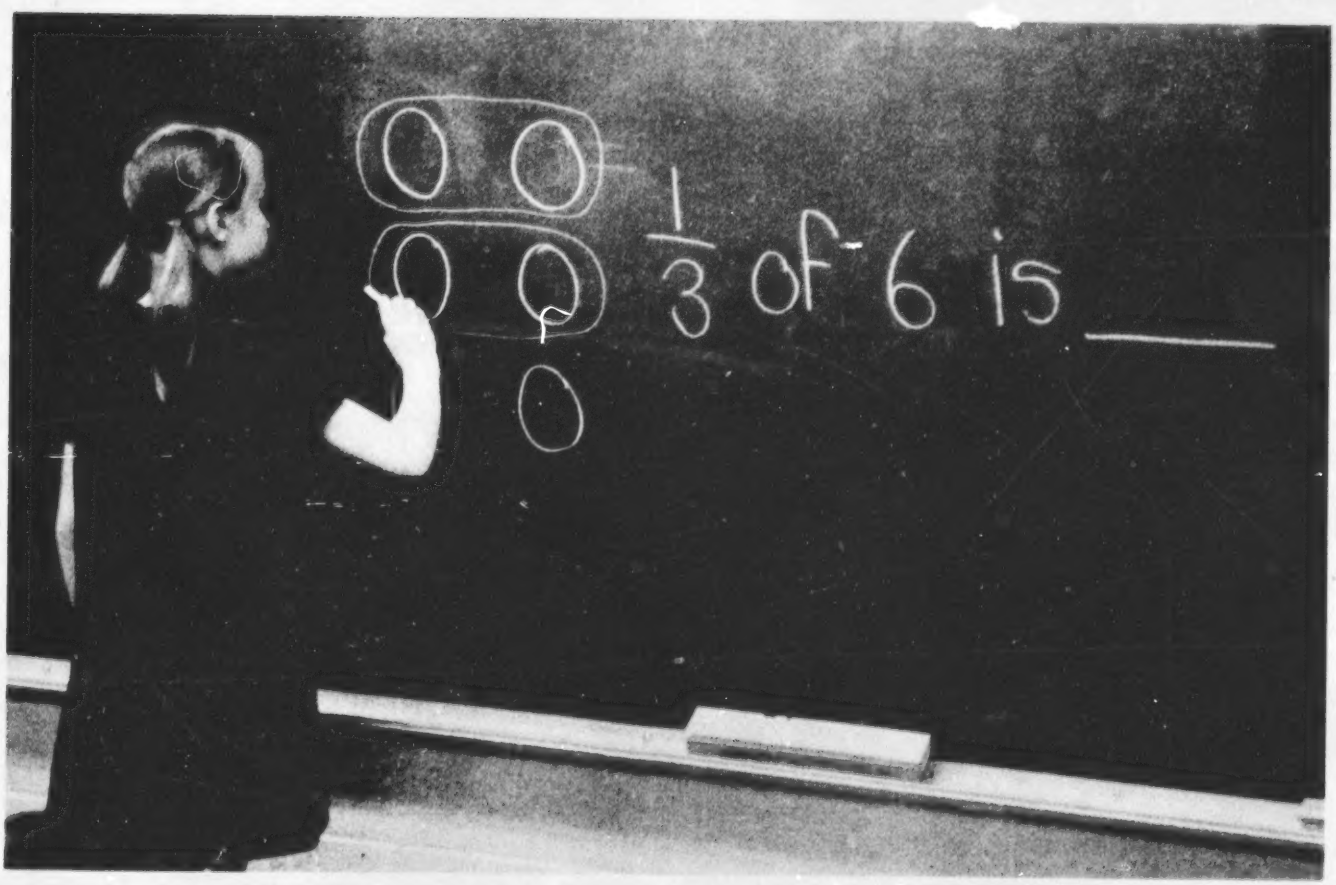
3. Puzzle Problem: If it takes two minutes to make each cut, how long will it take to cut a 10-foot pole into 10 equal pieces?

   $9 \times 2 = 18$ minutes

SECTION 4 SOME FRACTIONS

This section presents some simple fractions. Children have practical use for fractions at this age. In addition, fractions provide a background for the development of division in later units.

The children establish familiarity with fractions both by dividing a set into equivalent subsets and by finding fractional parts of geometric figures. For example, they will find $\frac{1}{3}$ of 12 by dividing a set of 12 members into three subsets of 4 members each. In another example they will find $\frac{1}{3}$ of the area of a rectangular shape by folding the shape into three equal parts.



Lesson 14: SOME SIMPLE FRACTIONS

This lesson introduces simple fractions, such as one-half, one-third, and one-fourth, by considering the separation of a set into a number of subsets containing the same number of members, that is, into equivalent subsets. For example, if there are three equivalent subsets, each is one-third of the original set, and we say that the number of elements in each subset is one-third of the number in the original.

The children should be encouraged to develop their own methods for breaking a set into equivalent subsets.

MATERIALS

- a flannel board with 24 objects representing cookies
- counters (optional)
- Worksheets 35, 36, 37, 38 and 39

PROCEDURE

Begin by reading the story "Jimmy's Favorite Cookies," pausing for class participation as indicated. The various divisions of 12 may be indicated with flannel board items, or with counters, or perhaps with drawings on the chalkboard. It is recommended that you always write a fraction with a horizontal line rather than with a slanted line, e.g. $\frac{1}{2}$ rather than 1/2, because the form with the horizontal line is easier for children to read and write.

JIMMY'S FAVORITE COOKIES

Jimmy came running home from school, and rushed into the house. "Mother, Mother!" he called. "Mother, where are you?"

"Here I am, Jimmy," his mother called from the kitchen. "What do you want to tell me?"

Jimmy ran into the kitchen. "Mother, some of the boys in my

room are going on a hike this afternoon, right away. I was wondering if I could take along something to eat. We're supposed to meet in front of our house, at 3 o'clock."

Jimmy's mother thought for a minute. "I don't know what I could give you. I haven't been to a store for almost a week."

"What about apples?" suggested Jimmy.

"No, you ate our last apple yesterday." Her face brightened. "Oh, yes, I know just the thing." She reached up to a high shelf, and brought out a white paper bag. "Mrs. Brown stopped in this morning, and left us a dozen peanut butter cookies. Now, I don't want you to be eating all of these before supper, but you can take them on one condition."

"What is that condition?" asked Jimmy, a little uneasily.

"You must divide these twelve cookies evenly among the boys who go on the hike," said Jimmy's mother.

"You had me worried," Jimmy replied. "That isn't such a bad condition. I promise that I will divide the cookies evenly among all the boys who come on the hike. I was expecting something much worse, like having to weed the garden, or scrub the porch."

Jimmy's mother laughed. "No, I won't take advantage of your wild craving for peanut butter cookies. Now, it's just about 3 o'clock. You'd better get out in front, and meet the other boys. Here are the cookies. Be sure to be home before suppertime and have a good hike."

"Goodbye, Mother, and thank you," Jimmy called, as he went out the door.

56

Jimmy waited in the front yard. He was looking forward to the hike, but right now he was thinking about the peanut butter cookies in the bag he was holding. He was getting hungrier and hungrier just thinking about them. Occasionally a delicious smell of peanut butter cookies would find its way out of the bag and float upward to his nose. Most mouth-watering of all smells!

At five minutes after three, none of the boys in his schoolroom had arrived. Jimmy was somewhat disappointed, and even impatient. But he had another thought. "If no one shows up for this hike, I will have all twelve of the peanut butter cookies for myself."

But at six minutes after three, before Jimmy had had too much chance to think about eating all twelve of the cookies, his friend Paul arrived, all ready for a good hike. Jimmy greeted Paul, but he was also thinking. "We'll have to divide the twelve cookies evenly into two shares. Paul and I will each get one-half of the cookies. That means we will each get ---"

HOW MANY COOKIES WILL EACH BOY GET? (Six. If a set of 12 things is divided evenly into two subsets, each subset will contain 6 things. We say a subset of 6 things is one-half of a set of 12 things, and we also say 6 is $\frac{1}{2}$ of 12.)

Jimmy was thinking that he could really fill up pretty well with six peanut butter cookies, and he was glad to have Paul's company on the hike. But at that moment their friend John came into the yard.

"Oh, oh," thought Jimmy, "Now we'll have to divide the cookies evenly into three shares. That means that Paul and John and I will each get one-third. We will each get ---"

HOW MANY COOKIES WILL EACH BOY GET NOW? (Four. Stress the fact that if a set is divided evenly into three subsets, each subset represents one-third of the total, and 4 is $\frac{1}{3}$ of 12.)

Before Jimmy could become completely used to the idea of having only four cookies, Henry came into the yard. Jimmy tried to hide his disappointment. He tried to act happy to see Henry arrive. But he was thinking, "Now, we'll have to divide the cookies evenly into four shares. We will each get one-fourth of the cookies. That means we'll each have ---"

($\frac{1}{4}$ of 12 is 3. Develop a discussion similar to the previous one.)

Jimmy loved peanut butter cookies, but he was also a good sport, and he was able to enjoy the boys gathered in his yard, even though he would now get only three cookies. And just then the twin brothers, Ed and Ned, came running up to the group. Jimmy thought to himself, "Now there are six of us. That is all who said they could come. That means we'll each get one-sixth of the cookies. And one-sixth of twelve is ---"

($\frac{1}{6}$ of 12 is 2. Develop a discussion similar to the previous one.)

By now it was quite clear to Jimmy that he would get no more than two of the cookies he was carrying but he made a big effort, and smiled as he greeted Ed and Ned.

Just as the boys were leaving the yard to begin their hike, Jimmy noticed that Ed and Ned were each carrying a white bag. "What do you have there?" he asked.

Ed and Ned answered together, "Our mother gave us each six peanut butter cookies to bring along. But we have to divide them evenly among all the boys on the hike."

NOW HOW MANY COOKIES WILL EACH BOY GET?
(Four. $\frac{1}{6}$ of 24 is 4.)

Have the children do Worksheets 35, 36, 37, 38 and 39.

Worksheet 35
Unit 17 Name _____

Here are 10 trees.

The trees are divided into [2] subsets.

There are [5] in each subset.

[5] is $\frac{1}{2}$ of 10.

Here are [18] triangles.

The triangles are divided into 3 subsets.

There are [6] in each subset.

[6] is $\frac{1}{3}$ of 18.

Worksheet 36
Unit 17 Name _____

Here are [6] cookies.

2 boys will share them.

Each will get $\frac{1}{2}$ of 6.

[3] is $\frac{1}{2}$ of 6.

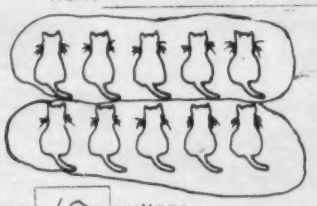
Jane had [12] tulips.

She planted them in 3 rows.

Each row has $\frac{1}{3}$ of 12.

[4] is $\frac{1}{3}$ of 12.

Worksheet 37
Unit 17 Name _____

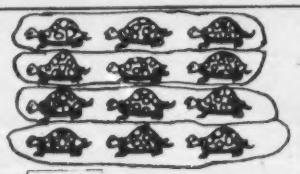


Here are [10] kittens.

Draw curves to divide the kittens into two subsets of the same number.

Each of the two subsets has [5] kittens.

$\frac{1}{2}$ of 10 is [5]



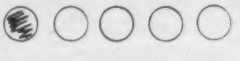
Here are [12] turtles.

Draw curves to divide the 12 turtles into 4 groups of the same number.

Each of the 4 groups has [3] turtles.

$\frac{1}{4}$ of 12 is [3]

Worksheet 38
Unit 17 Name _____

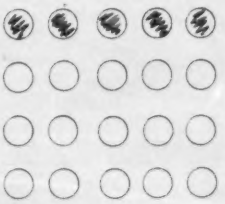


Color $\frac{1}{5}$ of this set.

$\frac{1}{5}$ of 5 is 1

Color $\frac{1}{2}$ of this set.

$\frac{1}{2}$ of 12 is 6



Color $\frac{1}{4}$ of this set.

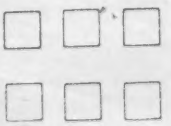
$\frac{1}{4}$ of 20 is 5

Color $\frac{1}{2}$ of this set.

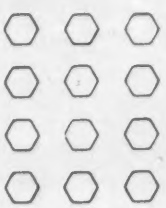
$\frac{1}{2}$ of 6 is 3

Worksheet 39
Unit 17 Name _____

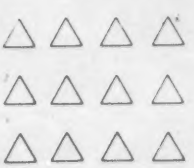
$\frac{1}{2}$ of 8 is 4



$\frac{1}{3}$ of 6 is 2



$\frac{1}{3}$ of 12 is 4



$\frac{1}{4}$ of 12 is 3

Lesson 15: FRACTIONAL PARTS OF PLANE FIGURES

This lesson gives another illustration of the meaning of certain simple fractions. A whole object is separated into 2 equivalent parts (halves), or 3 equivalent parts (thirds), or 4 equivalent parts (fourths).

MATERIALS

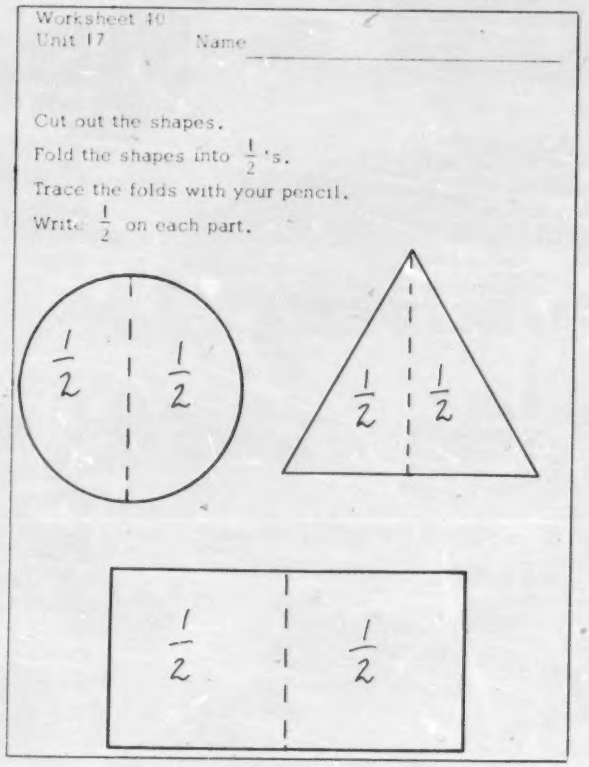
- scissors
- Worksheets 40, 41 and 42

PROCEDURE

Have the children cut out the three shapes on Worksheet 40. Have each child pick up his circular piece.

CAN YOU FOLD YOUR CIRCULAR PIECE INTO TWO PARTS OF EQUAL AREAS?

Fold your piece into halves along with the children. Show them that one part fits exactly over the other. Explain that each part has $\frac{1}{2}$ of the total area. After the children have completed the folding, have them draw a pencil line tracing the fold of the circle. Label each part $\frac{1}{2}$. Then have them fold each of the other shapes in half, trace the folds and label each part.


Worksheet 40
Unit 17 Name _____

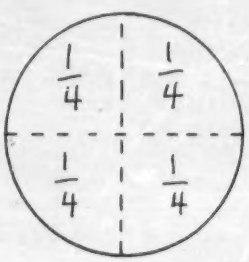
Cut out the shapes.
Fold the shapes into $\frac{1}{2}$'s.
Trace the folds with your pencil.
Write $\frac{1}{2}$ on each part.

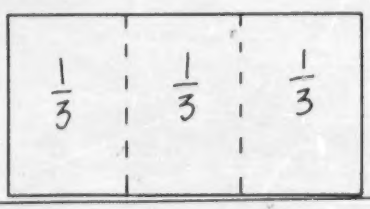
Do Worksheet 41 in a similar manner. Some children may be interested in using other folds and other shapes. You might suggest trying to fold a circle into thirds.

Have the children complete Worksheet 42.

Worksheet 41
Unit 17 Name _____

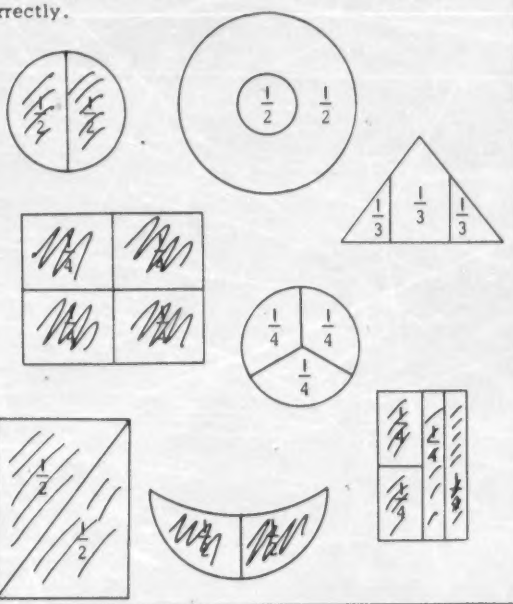
Cut out the shapes.
Fold the circular shape into $\frac{1}{4}$'s.
Write $\frac{1}{4}$ on each part.
Fold the rectangular shape into $\frac{1}{3}$'s.
Write $\frac{1}{3}$ on each part.





Worksheet 42
Unit 17 Name _____

The shapes below are divided into parts.
Color only the shapes where the parts are labeled correctly.



Lesson 16: HALVES ON THE NUMBER LINE

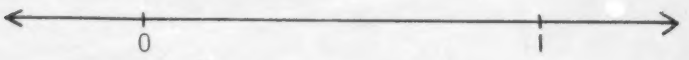
In this lesson points halfway between counting numbers are located and labeled on the number line.

MATERIALS

- three strips of paper about 18 inches long
- Worksheet 43

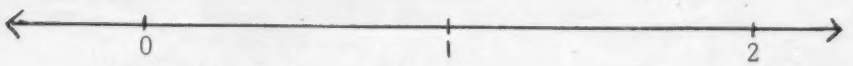
PROCEDURE

Draw a diagram similar to the following on the chalkboard. The distance between 0 and 1 should be about 18 inches.



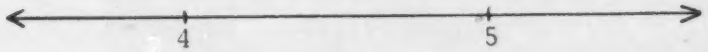
Ask the class to suggest ways to find one-half of the unit space between 0 and 1. One way is the following method, but they may prefer another method. Cut a strip of paper to fit between 0 and 1 on the number line, fold the strip in half, open it, and use the crease to mark the $\frac{1}{2}$ point on the number line.

Now draw this segment of the number line.



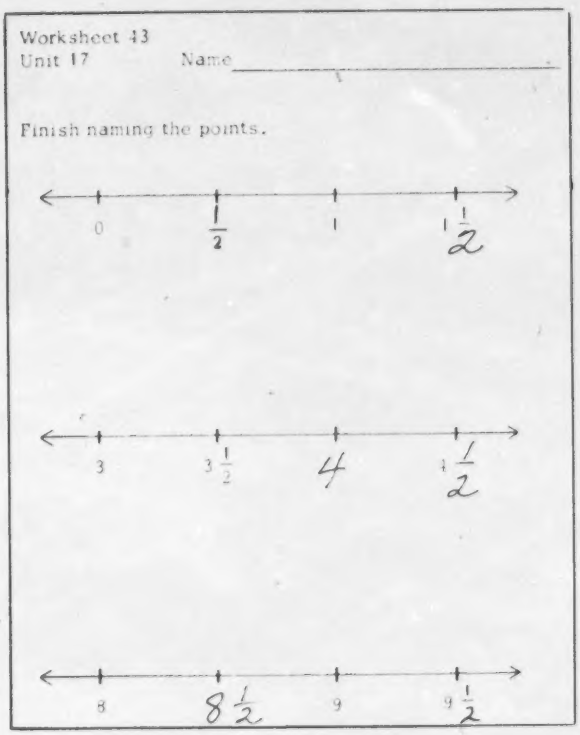
Have the class find the point halfway between 1 and 2. Tell the children that this point is $\frac{1}{2}$ unit space farther along the number line than 1; it is 1 and $\frac{1}{2}$ spaces from 0. Label the point 1 and $\frac{1}{2}$. Then show them that a shorter way to write this is $1\frac{1}{2}$.

Draw this segment of the number line.



Have the class find the point halfway between 4 and 5. Again label the point with "4 and $\frac{1}{2}$" and with $4\frac{1}{2}$.

Have the children complete the labeling of the points on the number lines on Worksheet 43.

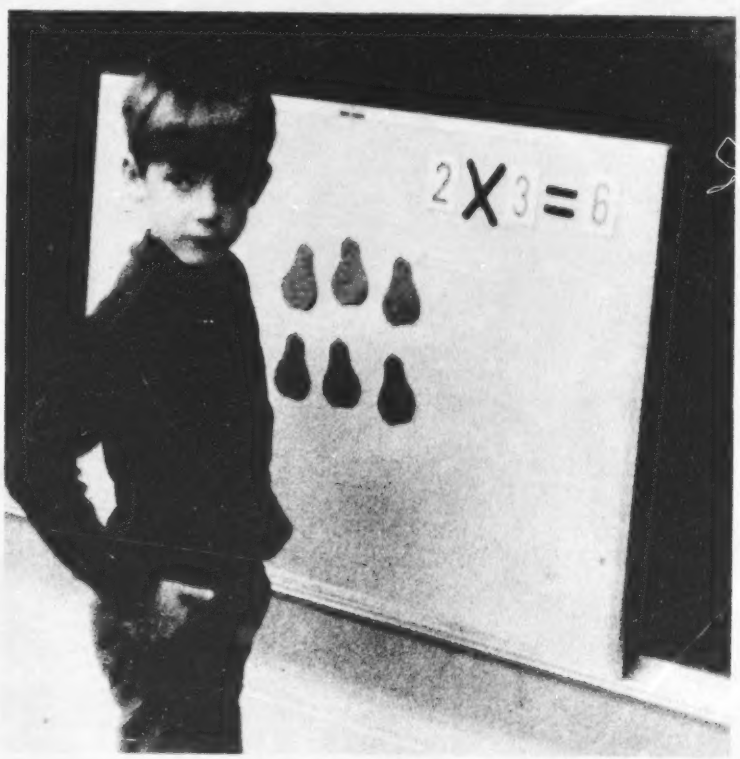


SECTION 5 REVIEW

When a child has completed the first four sections of this unit he should be able to perform the tasks listed below, although his performance on the fourth item may be imperfect. Only the integers 0 through 6 need be used in the multiplication problems.

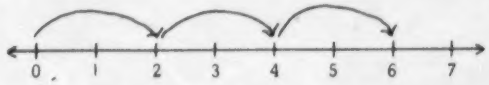
1. Interpret multiplication as repeated addition.
2. Show multiplication on parallel numbers lines of different scales.
3. Make rectangular arrays and use them to interpret multiplication.
4. Differentiate between the operations of addition and multiplication and use them correctly in simple situations.
5. Interpret the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ by dividing sets into equivalent subsets and areas into equal parts.

The items in the list are reviewed and extended in Unit 20. However, during the period between Unit 17 and Unit 20, you may wish to reinforce the children's concepts by using the Review Worksheets 44 through 51.

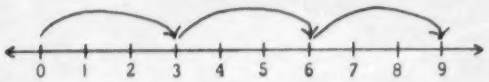


Worksheet 44
Unit 17 Name _____

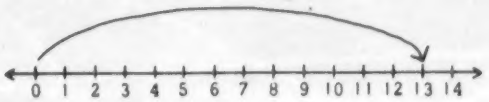
Finish each equation.
Show each problem on a number line.



3 x 2 = 6



3 x 3 = 9



1 x 13 = 13

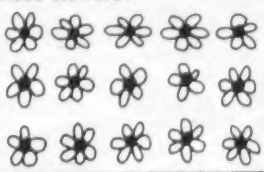
Worksheet 45
Unit 17 Name _____

Draw a 3 x 6 rectangular array.
The array has 18 members.
This shows that 3 x 6 = 18

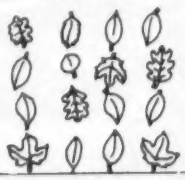
Draw a 6 x 3 rectangular array.
The array has 18 members.
This shows that 6 x 3 = 18

Worksheet 46
Unit 17 Name _____

Color $\frac{1}{3}$ of these flowers.



Color $\frac{1}{4}$ of these leaves.



Color $\frac{1}{3}$ of these flowers.

Worksheet 47
Unit 17 Name _____

Finish these charts.
Use your counters to help you.

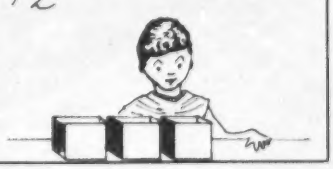
+	2	4
3	5	7
5	7	9

x	2	4
3	6	12
5	10	20

Write the equation that answers this problem.
Mark the answer on one of the charts.

Bill has 3 boxes of books.
Each box has 4 books in it.
How many books does he have?

3 x 4 = 12



Worksheet 48
Unit 17 Name _____

Write the equation that answers each question.
Use your number lines or counters to help you.

1. Sam, Ted, and Sally are making a rock collection.
   Ted brought 12 rocks.
   Sam brought 8 rocks.
   How many rocks did they bring together?

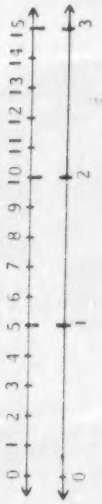
   $12 + 8 = 20$

2. Sally also brought her rocks.
   They put all the rocks on the table.
   They made 5 rows of rocks.
   Each row had 5 rocks.
   How many rocks were on the table?

   $5 \times 5 = 25$

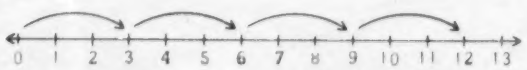
Worksheet 49
Unit 17 Name _____

Write in the missing numerals.



Spaces on top line	0	5	10	15	20
Spaces on bottom line	0	1	2	3	4

Worksheet 50
Unit 17 Name _____

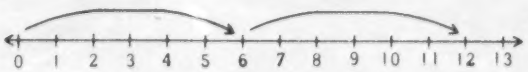


Write an addition equation about the steps on this number line.

$3+3+3+3=12$

Write a multiplication equation about the same steps.

$4 \times 3 = 12$



Write an addition equation about the steps on this number line.

$6 + 6 = 12$

Write a multiplication equation about the same steps.

$2 \times 6 = 12$

Worksheet 51
Unit 17 Name _____

Write the missing numerals.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

1 x 7 = 7
2 x 7 = 14
3 x 7 = 21
4 x 7 = 28

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21

1 x 3 = 3
2 x 3 = 6
3 x 3 = 9
4 x 3 = 12
5 x 3 = 15
6 x 3 = 18
7 x 3 = 21

Father Kangaroo

Katie Kangaroo

Joey Kangaroo

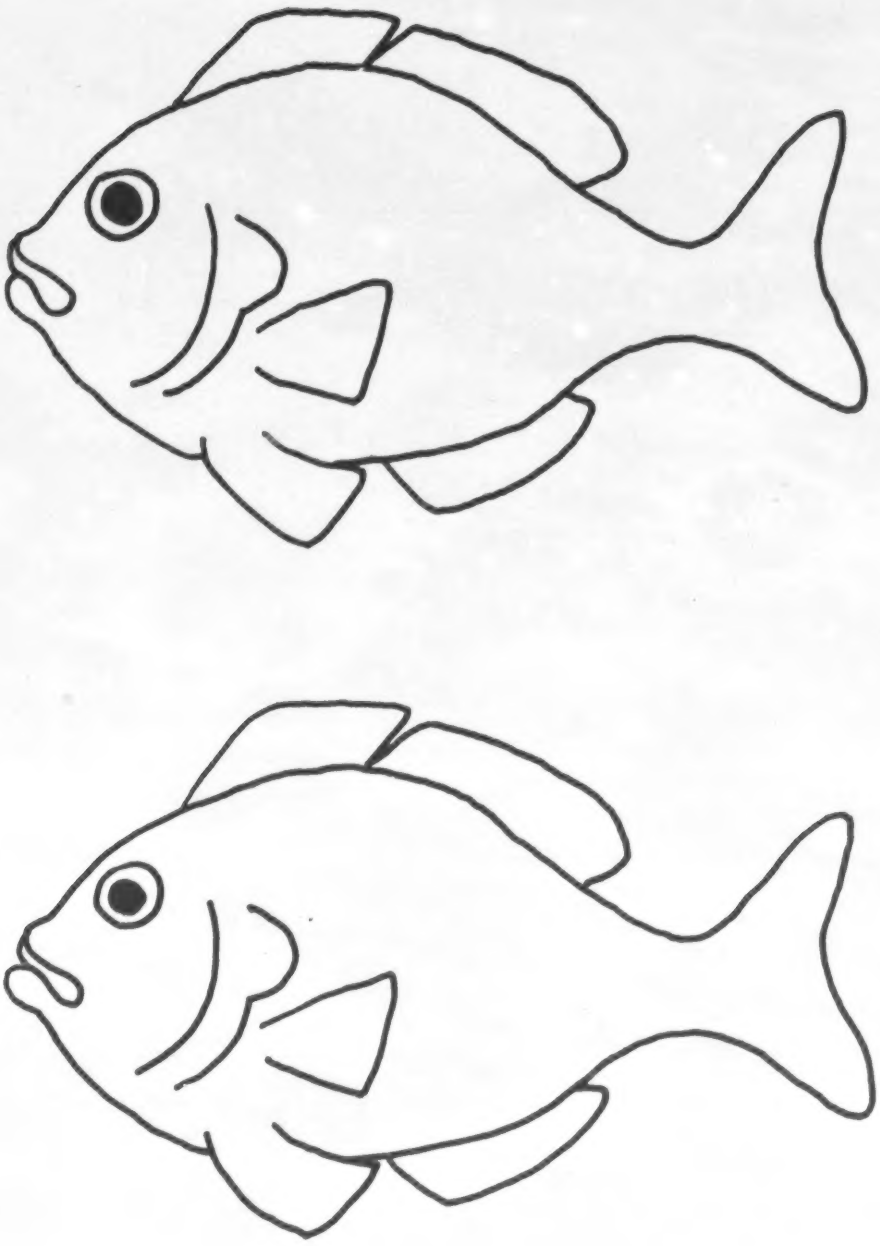
Kangaroo Cutouts for Lesson 1

61

Chuck Kangaroo

Mother Kangaroo

69



Fish patterns for Lesson 8